

## ACCURATE IDENTIFICATION OF BH-LOOPS FROM MEASUREMENT USING WIDE RING SPECIMEN

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**Abstract**

An analytical formulation is developed for identifying the hysteretic property of magnetic material from the measured current-flux property of a ring specimen. A computational example shows that the proposed method can reconstruct the hysteretic property more accurately than conventional approximations.

**1 Introduction**

The magnetic property of core material is often measured using a ring specimen. It is recommended that the ratio of the outer radius to the inner one should be equal to or less than 1.1 [1, 2] to avoid the field nonuniformity inside the ring specimen. However, it is sometimes difficult to manufacture a narrow ring sample. Several methods have been proposed to reconstruct an accurate BH curve taking account of the nonuniformity [2]. However, its numerical procedure is often cumbersome and the application to identify hysteresis loops is not straightforward.

This article proposes an analytical formulation for accurately identifying BH loops using a wide ring specimen.

**2 Identification of BH curves****2.1 Conventional identification of BH curves**

The magnetic property is supposed to be  $B = f(H)$ . This property is identified from measurement using a ring specimen having a rectangular cross-section with the inner radius  $a$ , the outer radius  $b$ , thickness  $w$ , and the number of primal and secondary windings  $N_1$  and  $N_2$ . When the excitation current is  $I$ , the magnetic field at the radius  $r$  is given as  $H = N_1 I / 2\pi r = i / r$ , where  $i = N_1 I / 2\pi$ . The flux linkage  $\Phi = wN_2 \int_a^b f(i/r) dr$  is obtained by measurement. Hence,  $\varphi = \Phi / wN_2$  is given as the function of  $i$ , which is written as

$$\varphi(i) = \int_a^b f(i/r) dr. \quad (1)$$

The averaged  $B$  is given by  $B_{ave} = \varphi(i) / (b-a)$ .

By setting the average path length  $\pi(a+b)$  or  $2\pi(b-a) / \log(b/a)$ , averaged  $H$  corresponding to  $B_{ave}$  is given as

$$H_{avep} = 2i / (a+b). \quad (2)$$

$$H_{avef} = i \log(b/a) / (b-a). \quad (3)$$

Conventionally, relation  $(H_{avep}, B_{ave})$  or  $(H_{avef}, B_{ave})$  has been used as the core property. This article calls the former "mean path approximation" and the latter "mean field approximation".

**2.2 Accurate Identification of BH curves**

The differentiation of Eq. (1) gives

$$d\varphi(i)/di = \int_a^b df(i/r)/di dr = \int_a^b f'(i/r)/r dr. \quad (4)$$

Using  $H = i/r$ , Eqs. (1) and (4) are rewritten as

$$\varphi(i) = \int_{H_a}^{H_b} f(H) i/H^2 dH, \quad (5)$$

$$d\varphi(i)/di = [f(H)/H]_{H_b}^{H_a} + \int_{H_b}^{H_a} f(H)/H^2 dH. \quad (6)$$

where  $H_a = i/a$  and  $H_b = i/b$ . From Eqs. (5) and (6)

$$f(H_a) / H_a - f(H_b) / H_b = d\varphi(i)/di - \varphi(i) / i \quad (7)$$

is obtained. By setting

$$F(i) = d\varphi(i)/di - \varphi(i) / i \quad (8)$$

$$c = a/b = H_b/H_a (< 1) \quad (9)$$

Eq. (7) is rewritten as

$$f(H_a) / H_a = f(cH_a) / cH_a + F(i). \quad (10)$$

Replacing  $i$  by  $c^n i$  in Eq. (10), one obtains  $f(c^n H_a) / c^n H_a = f(c^{n+1} H_a) / c^{n+1} H_a + F(c^n i)$ . Consequently,

$$\begin{aligned} f(H_a) / H_a &= f(c^2 H_a) / c^2 H_a + F(c i) + F(i) \\ &= \dots = \mu_1 + \sum_{n=0}^{\infty} F(c^n i). \end{aligned} \quad (11)$$

is derived, where  $f(H) / H \rightarrow \mu_1$  ( $H \rightarrow 0$ ).

**2.3 Identification of hysteresis loops**

The hysteretic BH curve depends on the amplitude  $H_m$  of the magnetic field as

$$B = f(H, H_m). \quad (12)$$

The amplitude of the exciting current is denoted by  $I_m$  and  $i_m = N_1 I_m / 2\pi$ . Using a constant  $\alpha$  ( $-1 \leq \alpha \leq 1$ ), the exciting current  $i$  is written as  $i = \alpha i_m$ .

The flux linkage  $\Phi = wN_2 \int_a^b f(i/r, i_m/r) dr$  is obtained by measurement. Hence,  $\Phi / wN_2 = \int_a^b f(i/r, i_m/r) dr$  is given as the function of  $i_m$  and  $i = \alpha i_m$ , which is written as

$$\int_a^b f(\alpha i_m/r, i_m/r) dr = \varphi(\alpha i_m, i_m) = \varphi_\alpha(i_m). \quad (13)$$

By setting

$$F_\alpha(i_m) = d\varphi_\alpha(i_m)/di_m - \varphi_\alpha(i_m) / i_m \quad (14)$$

a similar relation to Eq. (11)

$$f(\alpha H_a, H_a) / H_a = \mu_{\alpha 1} + \sum_{n=0}^{\infty} F_\alpha(c^n i_m). \quad (15)$$

is derived, where  $H_a = i_m/a$  and  $f(\alpha H_a, H_a) / H_a \rightarrow \mu_{\alpha 1}$  ( $H_a \rightarrow 0$ ).

By changing  $\alpha$  within  $-1 \leq \alpha \leq 1$ , hysteretic BH curves  $B = f(H, H_m)$  can be obtained.

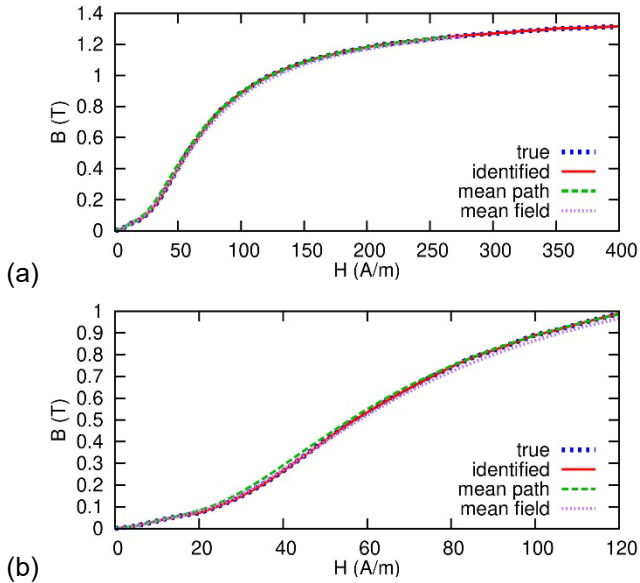


Figure 1: Normal magnetization curve: (a) overall and (b) enlarged views.

### 3 Computational results

Measured BH loops of non-oriented steel sheet JIS: 50A470 are used as the reference property (12). Setting  $c = 0.5$ , flux  $\varphi_\alpha(i_m)$  is calculated as Eq. (13). Even though Eq. (15) is an analytical expression, its calculation requires numerical techniques. For example, after obtaining the hysteretic property  $\varphi(i, i_m)$ , it is necessary for the identification procedure to represent  $\varphi(i, i_m)$  by a

hysteresis model. We used the play model [3] to represent  $i$ - $\varphi$  loops accurately for arbitrary  $i_m$  and  $\alpha i_m$ .

The reconstructed normal magnetization curve and BH loops are shown in Figs. 1 and 2. The proposed identification method gives more accurate BH curves than the conventional method using the mean path and field approximations. In addition, the obtained ranges of  $H_{avep}$  and  $H_{avef}$  are narrow because they are smaller than field  $H_a$  at  $r = a$ . When  $F(i)$  or  $F_\alpha(i)$  is positive (negative), the mean path (field) approximation becomes inaccurate where  $B > 0$ . When  $c = 0.5$ , the summation in (11) and (15) up to  $n = 10$  is sufficient. When  $c$  is near 0.9, the convergence of summation is slow, and conventional approximations have sufficient accuracy.

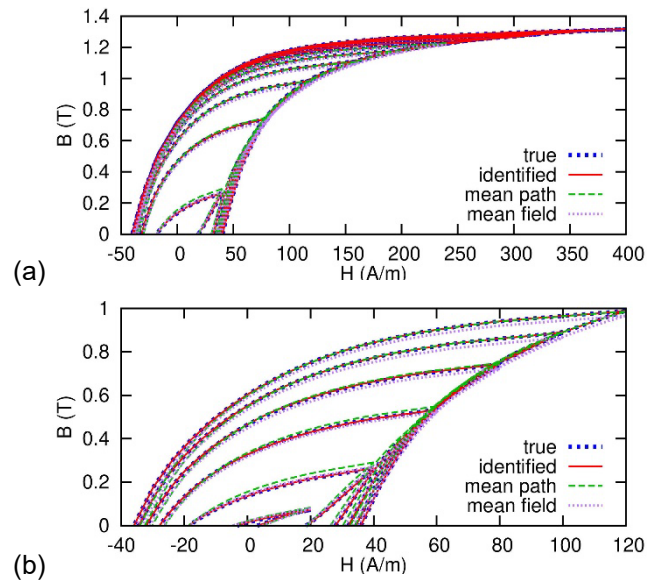


Figure 2: BH loops: (a) overall and (b) enlarged views.

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